

# 1.2 Polynomials

## Question Paper

Course	CIEA Level Maths
Section	1. Algebra & Functions
Topic	1.2 Polynomials
Difficulty	Very Hard

**Time allowed:** 80  
**Score:** /63  
**Percentage:** /100

**Question 1a**

(a) Expand and simplify  $(x + y)(x - y)(y - x)(-x - y)$ .

[2 marks]

**Question 1b**

(b) A cuboid has a length of  $(2x - 3y + 3)$  units, a width of  $(2x + 3y - 3)$  units, and a height of  $(x - y)$  units. Find an expression for the volume of the cuboid in terms of  $x$  and  $y$ .

[2 marks]

**Question 2**

Given that  $(ax + by)(2x + y)(x - 3y) = 8x^3 + cx^2y + dxy^2 - 9y^3$ , where  $a, b, c$  and  $d$  are constants, find the values of  $a, b, c$  and  $d$ .

[3 marks]

**Question 3**

Factorise completely  $x^5y - xy^5$ .

[3 marks]

**Question 4**

Divide  $4x^4 - 37x^2 + 9$  by  $(2x - 1)$ .

[3 marks]

**Question 5a**

$$f(x) = 6x^4 + 7x^3 - 27x^2 - 28x + 12$$

(a) Find the remainder when  $f(x)$  is divided by  $(2x + 3)$ .

[2 marks]

**Question 5b**

(b) Given that  $(x + 2)$  is a factor of  $f(x)$ , factorise  $f(x)$  completely.

[5 marks]

**Question 6a**

$$f(x) = 3x^4 + x^3 - 12x^2 - 49x - 15$$

(a) Show that  $f(x) = (3x + 1)(ax^3 + bx^2 + cx + d)$  where  $a, b, c$  and  $d$  are constants to be found.

[2 marks]

**Question 6b**

(b) Given that  $(x - 3)$  is a factor of  $f(x)$ , factorise  $f(x)$  completely.

[5 marks]

**Question 6c**

(c) Hence show that the equation  $f(x) = 0$  has exactly 2 real roots.

[2 marks]

**Question 7**

Given that 3 is a root of the equation  $2x^3 - x^2 - 11x - 12 = 0$ , prove that the equation has no other real roots.

[4 marks]

**Question 8a**

$$f(x) = 2x^4 - 15x^3 - 10x^2 + 105x + 98$$

(a) Show that  $f(-1) = 0$  and  $f(-2) = 0$ .

[1 mark]

**Question 8b**

(b) Hence, solve  $f(x) = 0$ .

[7 marks]

**Question 9**

Given that  $(2x - 5)$  is a factor of the function

$$f(x) = 2x^3 + kx^2 - 11x - 60$$

find the value of  $k$  and fully factorise  $f(x)$ .

[4 marks]

**Question 10**

Show that  $(9x^2 - 4)$  is a factor of  $9x^4 - 40x^2 + 16$  and hence find all the real solutions to the equation  $9x^4 - 40x^2 + 16 = 0$ .

[5 marks]

**Question 11a**

(a) Show that  $(ax - 2)$  is a factor of  $3ax^2 + (a - 6)x - 2$ .

[2 marks]

**Question 11b**

(b) Given that  $x = -\frac{1}{a-4}$  is a root of  $3ax^2 + (a-6)x - 2$ , find the value of  $a$ .

[3 marks]

**Question 12a**

For a polynomial  $f(x)$ , the Remainder Theorem states that

When  $f(x)$  is divided by  $(ax - b)$  the remainder is  $f\left(\frac{b}{a}\right)$ .

(a) Use the Remainder Theorem to find the remainder when  $8x^3 + 6x^2 - x - 2$  is divided by  $(2x + 1)$ .

[2 marks]

**Question 12b**

(b) Work out the remainder when  $6x^2 - x - 2$  is divided by  $(2x + 1)$ .

[2 marks]



**Question 13**

When  $2x^3 + (a + b)x^2 + (a - b)x - 3$  is divided by  $x + 4$  the quotient is  $2x^2 + (2a + 3)x + (2b - 5)$  and the remainder is  $c$ .

Find the values of  $a, b$  and  $c$ .

[4 marks]